Optimization Methods in Finance

Lecture 5

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Outline

1. CVX
2. Markowitz
3. Sharpe-Ratio – Alternative Way
4. Markowitz Model on Real Data
5. Rebalancing Portfolio
6. The Black – Litterman Model (and Investor’s View)
Problem 1.

\[
\min_{x \in \mathbb{R}^n} \sqrt{x^T Q x}
\]

subject to

\[
e^T x = 1, \quad \mu^T x \geq R, \quad x \geq 0.
\]
Problem I.

$$\begin{align*}
\text{min} & \quad \sqrt{x^T Q x} \\
\text{subject to} & \quad e^T x = 1, \quad \mu^T x \geq R, \quad x \geq 0.
\end{align*}$$

**Simple Trick**

$$\begin{align*}
\text{min} & \quad x^T Q x \\
\text{subject to} & \quad e^T x = 1, \quad \mu^T x \geq R, \quad x \geq 0.
\end{align*}$$
Problem II.

\[
\begin{align*}
\min_{x \in \mathbb{R}^n} & \quad \sqrt{x^T Q x} - \alpha \mu^T x \\
\text{subject to} & \quad e^T x = 1, \quad x \geq 0.
\end{align*}
\]
Markowitz

Sharpe Ratio – Alternative Way
Markowitz Model on Real Data
Rebalancing Portfolio

The Black – Litterman Model (and Investor’s View)
LET ME GET THIS STRAIGHT...
YOU WANT TO OFFER US FINANCIAL ADVICE?
Sharpe-Ratio – Alternative Way

Matlab
Markowitz Model on Real Data

- Fetching data from Yahoo Finance
- Estimating $\mu, \Sigma$ from data
- Plotting Efficient Frontier (the most easy part)
- Choosing Portfolio
- Comparison with S&P 500
"Sorry to disappoint you. All my money went to Greece."
Rebalancing Portfolio

- Additional constraint and penalty
- Comparison with S&P 500
"HEY! WHY DON'T WE JUST SAY WE HAVE NINETY-ONE PER CENT FULL EMPLOYMENT?"
The Black – Litterman Model

- Motivation
- Formulation
- Matlab
The Black – Litterman Model

- combine investor’s view with the market equilibrium
- developed in 1990 at Goldman Sachs by Fischer Black and Robert Litterman, and published in 1992
- seeks to overcome problems that institutional investors have encountered in applying modern portfolio theory in practice. The model starts with the equilibrium assumption that the asset allocation of a representative agent should be proportional to the market values of the available assets, and then modifies that to take into account the 'views' (i.e., the specific opinions about asset returns) of the investor in question to arrive at a bespoke asset allocation
The Black – Litterman Model

- Distribution of expected return $\mu$ is product of two multivariate normal distributions
- 1st - return at market equilibrium with mean $\pi$ and covariance $\tau \Sigma$
- 2nd - represents investor view about $\mu_i$’s

$$P\mu = q + \epsilon$$

where $\epsilon \sim \mathcal{N}(0, \Omega)$
Theorem - do NOT need to remember!

Let $X_1 \sim \mathcal{N}(\pi, \Sigma)$ and $X_2 \sim \mathcal{N}(\mu, \Omega)$ then

$$\mathbb{E}[X_1 \cdot X_2] = \Xi \cdot (\Sigma^{-1}\pi + \Omega^{-1}\mu)$$

where $\Xi = (\Sigma^{-1} + \Omega^{-1})^{-1}$. 

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**Black – Litterman Model**

$X_1 \sim \mathcal{N}(\pi, \tau \Sigma), \ X_2 \sim \mathcal{N}(P^+q, P^+\Omega(P^+)^T)$

Final $\bar{\mu}$ for the mean-variance optimization is given by

$$\bar{\mu} = \left[ (\tau \Sigma)^{-1} + P^T \Omega^{-1} P \right]^{-1} \left[ (\tau \Sigma)^{-1} \pi + P^T \Omega^{-1} q \right] \quad (8.7)$$
FANNIE MAE BAILOUT...
FREDDIE MAC BAILOUT...
MEDICARE BAILOUT...
SOCIAL SECURITY...
BAILOUT...

...WHAT HAPPENS WHEN
THE BILL ARRIVES??

NOT MY
PROBLEM, KID...
I PLAN TO
BAIL OUT
BEFORE
THEN.